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HEAT EXCHANGE OF AN IMMOVABLE FILTERED LAYER WITH IMMERSED SURFACES IN A TWO-COMPONENT MODEL OF HEAT TRANSFER

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Solutions for the problem of steady-state heat transfer in an immovable filtered layer with immersed surfaces are presented. Use is made of versions of a two-component homogeneous model differing in the ways of considering heat exchange between the layer components and the immersed surfaces. The results predicted by both versions are compared to one another and to experimental data, and the range of applicability is identified for each of them. Relations describing heat exchange of the layer components with a staggered tube bundle are given.

One of the important problems of the theory of dispersed media, which are heterogeneous systems, is a formulation of the heat-transfer models which fairly correctly describe real processes and are suitable for thermal calculations of various devices (chemical catalytic reactors, apparatuses for thermal treatment of dispersed materials, heat storage batteries, etc.). Diversity of structural, geometric, and operating characteristics of the dispersed systems and a simultaneous action of different transfer mechanisms make it reasonable to employ a formalized description based on a continuum approximation. The description is valid given that the characteristic internal scale of the medium is much smaller than the temperature field scale. Within the framework of such an approach, each component of the dispersed medium is regarded as a continuum with effective thermophysical characteristics, and their interaction is taken into consideration by appropriate transfer coefficients. The problem gets complicated when heat-transfer surfaces, providing a required temperature mode, are immersed in the dispersed medium.

The current paper reports a two-component continuum model of heat transfer with reference to one of the variants of dispersed systems, viz., to an immovable blown-through layer with immersed heat transfer surfaces, as well as relations for a temperature distribution of components derived on that basis. Two model versions, differing in the way of taking into account the heat exchange with the immersed surfaces were compared. The heat transfer process was considered in a one-dimensional approximation, with allowance for the following mechanisms: longitudinal conduction in gas and solid components, characterized by the effective axial coefficients of thermal conductivity λ_g^* and λ_{sol}^*, respectively; convective transfer by a gas component; and intercomponent exchange, defined by the gas–particle heat transfer coefficient α_i. The first version (model 1), as in [1, 2], took account of the heat exchange with the immersed surfaces for each of the layer components with the aid of relevant heat transfer components α_g and α_s. The second version (model 2) adopted a known (for example, [3, 4]) assumption of a negligible particle-surface contact area, which enabled us to disregard the heat transfer of the solid component. It was assumed in both cases that the porosity distribution and the gas velocity are uniform over a layer cross section, the heat transfer surfaces are located uniformly throughout the layer, and the thermal conductivity and the heat transfer coefficients are invariable.

For the first model version, a system of equations describing the steady-state heat transfer has the form:

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$$\lambda_{\mathrm{g}}^{*}(1-\beta_1)\frac{d^2\vartheta_{\mathrm{g}}}{dx^2}-G_{\mathrm{g}}C_{p\mathrm{g}}\frac{d\vartheta_{\mathrm{g}}}{dx}-\alpha_{\text{м}}a(1-\beta_2)(\vartheta_{\mathrm{g}}-\vartheta_{\text{т}})-\alpha_{\mathrm{s}}^{\text{г}}F_{\mathrm{s}}\vartheta_{\mathrm{g}}=0, \tag{1}$$

$$\lambda_{\mathrm{sol}}^{*}(1-\beta_1)\frac{d^2\vartheta_{\mathrm{sol}}}{dx^2}+\alpha_{\text{м}}a(1-\beta_2)(\vartheta_{\mathrm{g}}-\vartheta_{\mathrm{sol}})-\alpha_{\mathrm{s}}^{\text{т}}F_{\mathrm{s}}\vartheta_{\mathrm{sol}}=0. \tag{2}$$

The boundary conditions at the layer entrance are given by

$$x=0,\ \vartheta_{\mathrm{g}}=\vartheta_{\mathrm{g}0},\ \vartheta_{\text{т}}=\vartheta_{\mathrm{sol}0}. \tag{3}$$

A limitedness of a temperature rise in the layer at infinity was taken as a supplementary boundary condition.

A solution to the problem (1)-(3) is obtained using the asymptotic method of perturbations by small parameters $\lambda_{\mathrm{sol}}^{*}$, treated in [5]. The temperature fields in the layer components are defined as

$$\Theta_{\mathrm{g}}=\exp(AX), \tag{4}$$

$$\Theta_{\mathrm{sol}}=\exp\left(-X\sqrt{\frac{(1+\gamma)d^2}{\varepsilon_{\text{м}}}}\right)-\frac{\exp\left(-X\sqrt{\frac{(1+\gamma)d^2}{\varepsilon_{\text{м}}}}\right)-\exp(AX)}{1+\gamma-\varepsilon_{\text{м}}A^2}, \tag{5}$$

where

$$A=\frac{\mathrm{Pe}_{\text{м}}^{*}}{2}-\sqrt{\frac{(\mathrm{Pe}_{\text{м}}^{*})^2}{4}+\frac{d^2(1+\gamma)(\alpha_{\mathrm{s}}^{\mathrm{g}}F_{\mathrm{s}}(1+\gamma)+\alpha_{\mathrm{s}}^{\mathrm{sol}}F_{\mathrm{s}})}{(\lambda_{\mathrm{g}}^{*}(1+\gamma)^2+\lambda_{\mathrm{sol}}^{*})(1-\beta_1)}}; \tag{6}$$

$$\mathrm{Pe}_{\text{м}}^{*}=\frac{G_{\mathrm{g}}C_{p\mathrm{g}}(1+\gamma)^2}{(\lambda_{\mathrm{g}}^{*}(1+\gamma)^2+\lambda_{\mathrm{sol}}^{*})(1-\beta_1)}; \tag{7}$$

$$\gamma=\frac{\alpha_{\mathrm{s}}^{\mathrm{sol}}F_{\mathrm{s}}}{\alpha_{\text{м}}a(1-\beta_2)}; \tag{8}$$

$$\varepsilon_{\text{м}}=\frac{\lambda_{\mathrm{sol}}^{*}(1-\beta_1)}{\alpha_{\text{м}}a(1-\beta_2)d^2}; \tag{9}$$

$$X=\frac{x}{d}. \tag{10}$$

The complexes γ and ε_i, and the modified Peclet number characterize heat flux relationships, specified by various heat transfer mechanisms.

When the intercomponent heat exchange in the layer is much larger than the heat transfer by heat conduction in the solid component and than the heat exchange of the solid component with the immersed surfaces, a temperature slip between the gas and particles virtually vanishes, and it is possible to employ a one-component model which considers the layer as a homogeneous medium with effective characteristics. Thus, the conditions for conversion of the two-component model to that one-component are as follows:

$$\gamma\ll 1, \tag{11}$$

$$\varepsilon_{\text{м}}\ll 1. \tag{12}$$

The assumption of a negligible contact area of the solid component particles and immersed surfaces, made in the second model version, allowed the heat exchange of the solid component with the surfaces to be ignored ($\alpha_{\mathrm{s}}^{\mathrm{sol}}=0$) in Eq. (2). In this case, a solution for the problem (1)-(3) is

$$\Theta_{\mathrm{g}}=\exp(RX), \tag{13}$$

$$\Theta_{\mathrm{sol}}=\exp\left(-X\sqrt{\frac{d^2}{\varepsilon_{\text{м}}}}\right)-\frac{\exp\left(-X\sqrt{\frac{d^2}{\varepsilon_{\text{м}}}}\right)-\exp(RX)}{1-\varepsilon_{\text{м}}R^2}, \tag{14}$$

where

$$R = \frac{\mathrm{Pe}^*}{2} - \sqrt{\frac{(\mathrm{Pe}^*)^2}{4} + \frac{\alpha_s F_s d^2}{(\lambda_g^* + \lambda_{sol}^*)(1-\beta_1)}}; \tag{15}$$

$$\mathrm{Pe}^* = \frac{G_g C_{pr} d}{(\lambda_g^* + \lambda_{sol}^*)(1-\beta_1)}. \tag{16}$$

When the heat exchange of the solid component with the immersed surfaces is negligible as compared to the intercomponent heat transfer (i.e., with $\gamma << 1$), and

$$\alpha_s = \alpha_s^g + \alpha_s^{sol}, \tag{17}$$

Eqs. (4) and (5) go then, correspondingly, into Eqs. (13) and (14), and the temperatures predicted from the two model versions are actually identical.

Calculations by the presented relations necessitate information regarding the heat transfer characteristics. Extensive data on the intercomponent heat-transfer coefficients are reported, for instance, in [6, 7]. A number of recommendations are known for computing the effective axial coefficients of thermal conductivity for the components of a densely packed blown-through layer (for example, [8, 9]). The data on the coefficients of heat transfer with immersed surfaces pertain to a layer as a whole, i.e., they are applicable only to the one-component model [10-14]. No information as to the heat transfer coefficients for components α_s^g and α_s^{sol} is available in the literature. To determine them, the authors worked out methods based on solving an inverse problem of heat transfer and performed experimental investigations.

Conditions in a test stand were set up in accordance with the assumptions under which the calculational relations were derived. Experiments were conducted with staggered bundles of tubes without finning. Monodispersed layers of glass or corundum spherical particles 2.95-9.3 mm in diameter were used. Characteristics of the immersed surfaces were the following: F_s = 23.4 m^2/m^3, D = 0.0335 m, β_1 = 0.5, and β_2 = 0.196. The layer was blown through by air of temperature 10-30°C. The data on the heat transfer with the staggered tube bundle for $25 \le \mathrm{Re} \le 1200$, $S_1/D = 2$, $S_2/D = 2$, $3.6 \le (S_1 - D)/d \le 11.36$, $3.6 \le (S_2 - D)/d \le 11.36$, and $28.11 \le \lambda_{sol}/\lambda_g \le 415$ are described, with a probable error of ±5.2%, by the following correlations, correspondingly, for the gas and solid components

$$\mathrm{Nu}^g = 3{,}08\,\mathrm{Re}^{0{,}46}(D/d)^{0{,}16}, \tag{18}$$

$$\mathrm{Nu}^{sol} = 0{,}27\,\mathrm{Re}^{0{,}44}(D/d)^{0{,}15}(\lambda_{sol}/\lambda_{sol})^{0{,}33}. \tag{19}$$

These data enable predictions using the first model version (Eqs.(4) and (5)).

From the two model versions, the layer temperatures are calculated so as to be quantitatively compared at various Reynolds numbers and characteristics of the solid component. Furthermore, sensitivity of both model versions to the error, with which α_s^g and α_{sol} are determined, is evaluated. Discrepancy between the temperatures, computed by models 1 and 2, is estimated using the relative temperature differences, viz., $\delta_g = (t_{g1} - t_{g2})/t_{g1}$ for the gas component and $\delta_{sol} = (t_{sol1} - t_{sol2})/t_{sol1}$ for the solid component. In calculations, the range of parameters and the characteristics of the immersed surfaces corresponded to those given above. An entrance temperature of the blown-through air was taken to be 30°C. The quantities λ_g^* and λ_{sol}^*, employed in predictions, were defined in conformity with [8], α_i was determined by recommendations of [7], α_s^g and α^{sol} were obtained from experimental relations (18) and (19), and α_s was found from Eq. (17). Figure 1 presents temperature distributions of the components along the layer height for two modes, in accordance with model 1. Model 2 produces similar results with definite quantitative differences.

As the predicted results made clear, the Re number significantly affects δ_g and δ_{sol}. Thus, at d = 9.3 mm, λ_{sol}= 11 W/(m·K), and x = 0.50 m, the values of δ_g and δ_{sol} comprise 5.7 and 14.3% for Re = 50, whereas these equal 1 and 5.8% for Re = 350.

With increasing particle diameter, a rise in δ_g and δ_{sol} is observed. When the particle diameter varies from 2.95 up to 9.3 mm for Re = 100, λ_{sol} = 11 W/(m·K), and x = 0.17 mm, δ_g grows from 5.7 up to 3.3%, and δ_{sol} increases from 3.2 up to 10.5%.

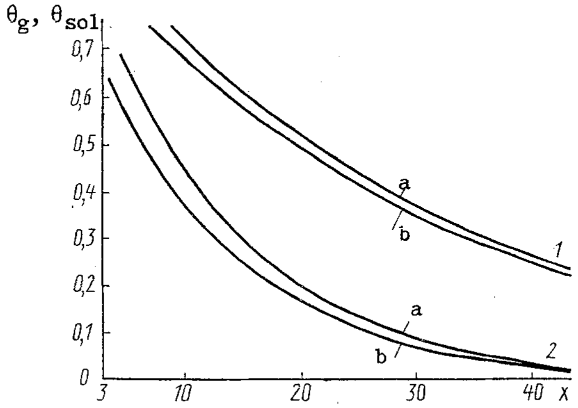

Fig. 1. Temperature fields of gas (a) and solid (b) components of a layer calculated from Eqs. (4) and (5) at d = 9.3 mm, λ_{sol} = 11 W/(m·K); 1) for Re = 500; 2) Re = 50.

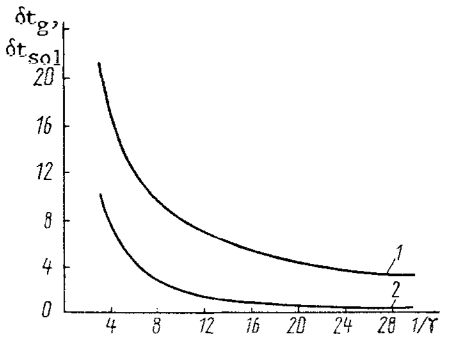

Fig. 2

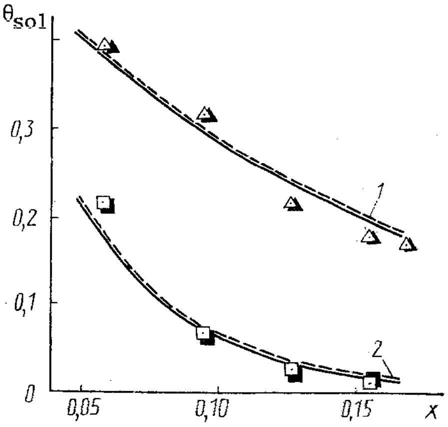

Fig. 3

Fig. 2. Effect of the complex $1/\gamma$ on discrepancy of temperatures corresponding to different models; 1) discrepancy of temperatures of a solid component calculated from Eqs. (5) and (4); 2) discrepancy of temperatures of a gas component calculated from Eqs. (4) and (13); x = 0.17 m.

Fig. 3. Comparison of experimental and calculated temperature fields in a layer at D = 0.0335 m; $S_1/D = 2$, $S_2/D = 2$, d = 3 mm, and dashed curves denote calculated temperatures of gas and particles, respectively; dots mark experimental temperatures of gas and particles.

As the thermal conductivity of the particle material increases, δ_g and δ_{sol} increase as well. Thus, with λ_{sol} ranging from 2 up to 11 W/(m·K) at Re = 100, d = 2.95 mm, and x = 0.17 m, δ_g increases from 0.5 up to 1.4%, and δ_T grows from 2.2 up to 3.3%.

Consequently, at specified geometric characteristics of the immersed surfaces, the discrepancy of the temperatures, calculated from models 1 and 2 is a function of the characteristics of the layer components, Reynolds number, and longitudinal coordinate. The complex γ (or $1/\gamma$), taking account of the indicated factors, can serve for a convergence criterion.

As is evident from Fig. 2, which plots the relative temperature differences vs the complex $1/\gamma$, δ_g and δ_{sol} increase noticeably with a decrease in this parameter. The values of δ_g and δ_{sol} in the region $1/\gamma > 30$ do not, in fact, vary with increasing $1/\gamma$.

Hence, a fulfillment of the condition (11) (actually, $1/\gamma > 30$ and $\gamma < 0.03$) provides a satisfactory agreement between the temperatures predicted from models 1 and 2, with consideration of relation (17). The above condition is satisfied at large Re numbers and small λ_{sol}, d, and x.

Figure 3 compares the temperature fields in the layer, computed by the model (1)-(3), to the experimental data obtained by the authors of the current paper (both model versions yield identical results for the experimental conditions). The comparison evidences a reasonable qualitative and quantitative agreement between the predicted results and experimental data, and, therefore, indicates that both versions of the model (1)-(3) adequately describe salient features of the heat transfer in the layer with immersed surfaces.

The calculations demonstrated that the errors, with which the coefficients α_s^g and α_s^{sol} are determined, have a substantial effect on the layer temperatures computed from both models. For example, for Re = 25, $\lambda_{sol} = 0.756$ W/(m·K), d = 2.95 mm, and x = 0.17 m, the component temperatures varied by 14% upon α_s^g and α_s^{sol} changing by 10%, and these varied by 31% upon α_s^g and α_s^{sol} altering by 20%. Consequently, inaccuracy in determining the coefficients of heat transfer of the components with the immersed surfaces can lead to noticeable errors in calculating the temperature fields. Experiments simplify significantly and produce a smaller error in obtaining the heat transfer coefficient α_s, rather than in estimating the heat transfer coefficients for each of the components α_s^g and α_s^{sol}. Therefore, with condition (11) observed, the second model version (Eqs. (13) and (14)) utilizing α_s is more convenient to use for predicting the temperature fields.

The presented data permit a substantiation of the requirements to accuracy of determining the coefficients of heat transfer with immersed surfaces and can be employed in devising the engineering methods to design bed devices. The problem is topical in view of a wide application of such devices and need for their optimization.

NOTATION

t, temperature; $\vartheta = t - t_s$, excess temperature; $\Theta = \vartheta/\vartheta_0$, dimensionless temperature; λ, thermal conductivity coefficient; ρ, density; C_p, specific heat; W_f, filtration rate; G, mass flow rate; D, diameter of immersed surfaces; F_s, area of immersed surfaces per unit volume of layer; S_1, S_2, longitudinal and transverse pitches of tubes; β_1, β_2, portions of cross section and volume occupied by immersed surfaces; x, longitudinal coordinate; d, particle diameter; a, specific surface of particles per unit volume of layer; $Re = W_f D/\nu$, Reynolds number; $Nu^g = \alpha_s^g D/\lambda_g$, $Nu^{sol} = \alpha_s^{sol} D/\lambda_{sol}$, Nusselt number. Subscripts: g, gas; sol, solid; s, surface; 0, entrance section.

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